Math 60 9.2 & 9.3 - 1st

Rational Exponents

Objectives

1) Write nth radicals using fractional exponents. $\sqrt[n]{x} = x^{1/n}$

2) Evaluate rational exponents

$$a^{1/n} = \sqrt[n]{a}$$

3) Evaluate rational exponents

$$a^{m/n} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$$

4) Review laws of exponents

$$a^n \cdot a^m = a^{n+m} \quad *$$

$$\frac{a^n}{a^m} = a^{n-m}$$

$$(a^n)^m = a^{nm} \quad *$$

especially these two

5) Simplify expressions containing fractional exponents by using the laws of exponents. and radicals.

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Recall exponent laws

$$a^n \cdot a^m = a^{n+m}$$

Notice that

$$\sqrt{a} \cdot \sqrt{a} = a$$

can be written using exponents

$$a^? \cdot a^? = a^1$$

where ? must be the same exponent

$$a^n \cdot a^n = a^1$$

$$a^{n+n} = a^1$$

$$a^{2n} = a^1$$

So these exponents must be

$$2n = 1$$

$$n = 1/2.$$

This means $a^{1/2} \cdot a^{1/2} = a^1$

OR, that $\boxed{a^{1/2} = \sqrt{a}}$.

We can demonstrate similarly that

$$\sqrt[3]{a} \cdot \sqrt[3]{a} \cdot \sqrt[3]{a} = a^1$$

means $\boxed{\sqrt[3]{a} = a^{1/3}}$

and $\boxed{\sqrt[4]{a} = a^{1/4}}$

So that $\boxed{\sqrt[n]{a} = a^{1/n}}$ for any positive integer n.

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Write with fraction exponent. Assume all variables are non-negative.

① $\sqrt[3]{9z}$

$= \boxed{(9z)^{1/3}}$

② $\sqrt[4]{\dfrac{a^3b}{7}}$

$= \boxed{\left(\dfrac{a^3b}{7}\right)^{1/4}}$

Evaluate, round 2 places. Assume variables are non-negative.

③ $16^{1/2}$

$= \sqrt[2]{16}$

$= \sqrt{16}$

$= \boxed{4}$

because $4^2 = 16$.

④ $(-8)^{1/3}$

$= \sqrt[3]{-8}$

$= \boxed{-2}$

because $(-2)^3 = -8$

(5) $-81^{1/2}$

exponent before mult

$= -(81^{1/2})$

$= -\sqrt{81}$

$= \boxed{-9}$

(6) $(-81)^{1/2}$

$= \sqrt{-81}$

$= \boxed{\text{not a real \#}}$

write as a radical.

(7) $p^{1/2}$

$= \boxed{\sqrt{p}}$

Recall exponent rules

(8) $(a^m)^n = \boxed{a^{n \cdot m}}$

one base, ( ), two exponents
$\Rightarrow$ multiply exponents.

Ex: $(2^3)^4 = (2 \cdot 2 \cdot 2)^4$

$= 2^4 \cdot 2^4 \cdot 2^4$

$= 2^{4+4+4}$

$= 2^{4 \cdot 3}$

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This means

9. $(a^m)^{1/n} = a^{m/n} = (a^{1/n})^m$

but $(a^m)^{1/n}$ also means $\sqrt[n]{a^m}$. and $(a^{1/n})^m = (\sqrt[n]{a})^m$

So:

$$a^{m/n} = (a^m)^{1/n} = (a^{1/n})^m$$

$$\sqrt[n]{a^m} = (\sqrt[n]{a})^m.$$

Fraction exponents mean <u>radicals</u>.

$a^{m/n}$

numerator is a more usual exponent

denominator is index of radical

Evaluate & round to two decimal places if needed.

10. $25^{3/2}$

$= (\sqrt{25})^3$

$= 5^3$

$= \boxed{125}$

It's usually easier to do radical first and then power

<u>OR</u> $= \sqrt{25^3}$

$= \sqrt{15625}$

$= \boxed{125}$

But it's legal (though difficult) to do the power first and then radical.

11. $125^{2/3}$

$= (\sqrt[3]{125})^2$

$= 5^2$

$= \boxed{25}$

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(12) $36^{3/2}$

$= (\sqrt{36})^3$ index 2 = denominator 2

$= 6^3$ evaluate from inside out

$= \boxed{216}$

Remember from Math 45:

$x^{-2} = \dfrac{1}{x^2}$

numerator negative exponent can be written as positive exponent in denominator

and

$\dfrac{1}{x^{-2}} = x^2$

denominator negative exponent can be written as positive exponent in numerator

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This rule about negative exponents also applies to negative rational (fraction) exponents:

$x^{-2/3} = \dfrac{1}{x^{2/3}} = \dfrac{1}{\sqrt[3]{x^2}}$

Notice that negative numbers can appear in three different locations. Each location means a specific thing, and cannot be confused for another location

$-2(-8)^{-2/3}$

ABOVE: exponent is negative means change the location of the base from the numerator to the denominator (or vice versa) first to get a positive exponent

IN FRONT: coefficient is negative

multiply by -2 last according to order of operations

INSIDE: base is negative

raise this number to the exponent

(13) $-36^{3/2}$

no parentheses — order of operations says exponents before multiply.

$= -\left(36^{3/2}\right)$

$= -\left(\sqrt{36}\right)^3$

denom = 2 = index of radical.

$= -(6)^3$

$= \boxed{-216}$

(14) $(-36)^{3/2}$

$= \left(\sqrt{-36}\right)^3 = \boxed{\text{not a real \#}}$

(15) $(-27)^{4/3}$

$= \left(\sqrt[3]{-27}\right)^4$

denominator 3 ⇒ cube root index 3.

parentheses ⇒ negative goes inside radical.

numerator ⇒ power 4

$= (-3)^4$

$(-3)^3 = -27$

$= \boxed{+81}$

skip: ~~(16)~~ $39^{3/4}$ Approximate to the nearest hundredth.

$= \left(\sqrt[4]{39}\right)^3$

39 is not a perfect fourth power but 39 is a positive #, so the $\left(\sqrt[4]{39}\right)^3$ can be calculated

$= 15.60624625$

$\approx \boxed{15.61}$

39^(3/4)

39^.75

$(\sqrt[4]{39})$^3

Three different but valid ways to use calculator.

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Evaluate.

(17) $49^{-1/2}$

$= \frac{1}{49^{1/2}}$ step 1: Write negative exponent as positive exponent by moving its base

$= \frac{1}{\sqrt{49}}$ step2: Write fraction exponent as radical.

$= \boxed{\frac{1}{7}}$ step3: Evaluate radical.

(18) $\frac{1}{64^{-2/3}}$ neg. exp. in denom.

$= 64^{2/3}$ write positive exp in numerator

$= \left(\sqrt[3]{64}\right)^2$ write as radical

$= (4)^2$ $4^3 = 64$ so $\sqrt[3]{64} = 4$

$= \boxed{16}$ exponent

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(19) $-2(-8)^{-2/3}$

step 1: If exponent is negative, rewrite as a positive exponent in the other part of the fraction.

$$\frac{-2(-8)^{-2/3}}{1} = \frac{-2}{(-8)^{2/3}}$$

Note: no exponent on -2, stays up.

Step 2: Write fractional exponent using a radical.

$$= \frac{-2}{(\sqrt[3]{-8})^2} \quad \text{or} \quad \frac{-2}{\sqrt[3]{(-8)^2}}$$

Recall: $a^{n/m} = \sqrt[m]{a^n}$ or $(\sqrt[m]{a})^n$

step 3: Evaluate the radical

$$= \frac{-2}{(-2)^2} \quad \text{or} \quad \frac{-2}{\sqrt[3]{64}}$$

$$= \frac{-2}{4} \qquad \frac{-2}{4}$$

step 4: Multiply by -2 (simplify the fraction)

$$= \boxed{-\frac{1}{2}}$$

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Extra practice

Simplify

(20) $(-81)^{1/2}$ neg base = radicand

$= \sqrt{-81}$

$=$ $\boxed{\text{not a real \#}}$

(21) $-25^{1/2}$ neg coef = multiply

$= -\sqrt{25}$

$= \boxed{-5}$

(22) $-81^{1/4}$ neg coef = multiply

$= -\sqrt[4]{81}$

$= -\sqrt[4]{3^4}$

$= \boxed{-3}$

(23) $(-81)^{1/4}$ neg base = radicand

$= \sqrt[4]{-81}$

$=$ $\boxed{\text{not a real \#}}$

(24) $25^{3/2}$

$= (\sqrt{25})^3$

$= 5^3$

$= \boxed{125}$

(25) $-25^{3/2}$ neg coef = multiply

$= -(\sqrt{25})^3$

$= -5^3$ exp before mult.

$= \boxed{-125}$

(26) $(-25)^{3/2}$

$= (\sqrt{-25})^3$

$=$ $\boxed{\text{not a real number}}$

(27) $121^{-1/2}$ ← neg exp, move to denominator

$= \dfrac{1}{121^{1/2}}$

$= \dfrac{1}{\sqrt{121}}$

$= \boxed{\dfrac{1}{11}}$

(28) $\dfrac{1}{49^{-3/2}}$ ← neg exp, move to numerator

$= 49^{3/2}$

$= (\sqrt{49})^3$

$= 7^3$

$= \boxed{343}$